What's all this VLBI stuff, anyway?

Tom Clark

NVI / NASA Goddard Space Flight Center

mailto: K3IO@verizon.net

IVS TOW Workshop

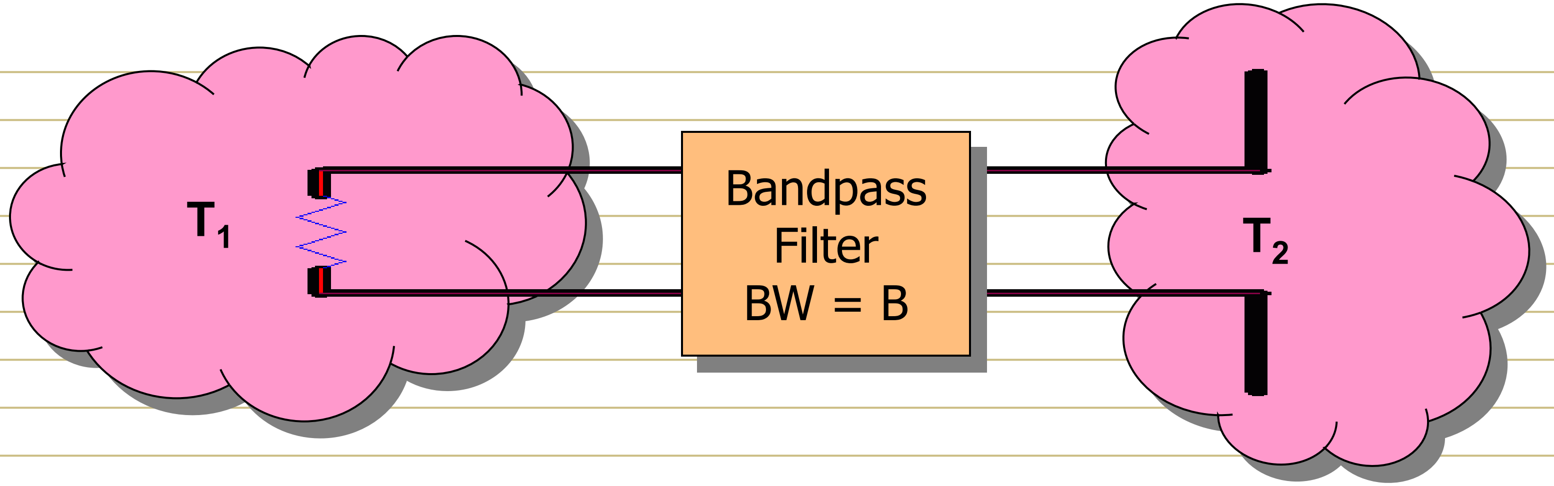
Haystack – April 30, 2007

The Topics for Today:

1. Some Fundamentals of Radio Astronomy
   - Noise, Temperature = Power
   - Janskys, Flux Density & Sensitivity
2. Some Fundamentals of Interferometry
   - What is an interferometer?
   - Resolution & Spatial Frequency
   - Heisenberg's Uncertainty Principle
   - The U-V Plane and Aperture Synthesis
3. How is VLBI Different?
   - Breaking the Wires and shipping the bits
   - Quasars and similar beasties
   - Closure Phase
   - Group Delay

Note: **Some important concepts are marked like this**

Some Simple Thermodynamics

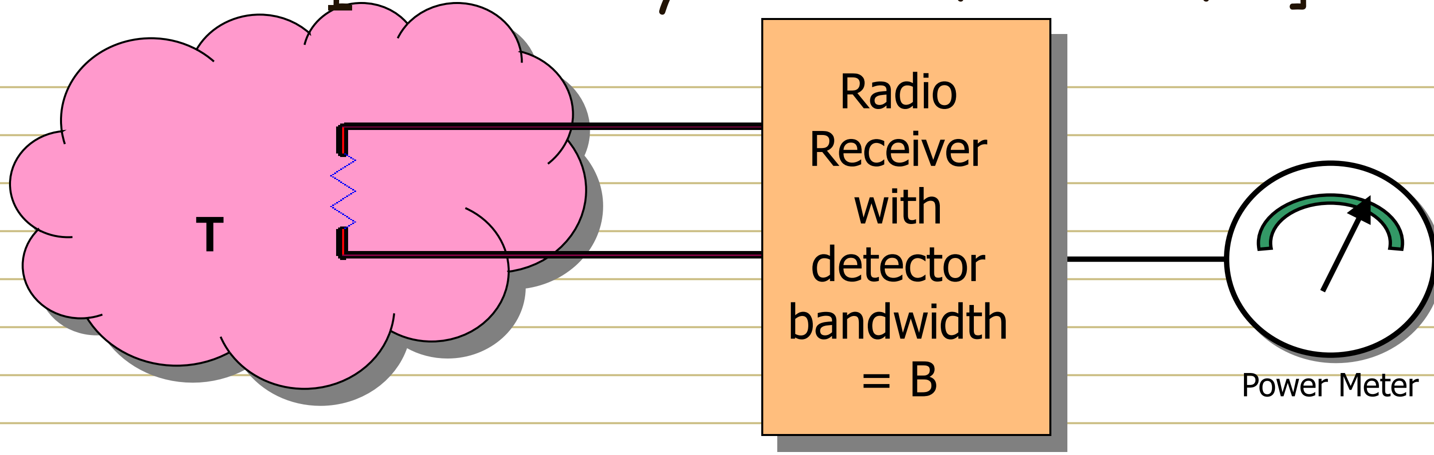


- Consider two isolated universes at two different temperatures, T_1 & T_2 , and let $\Delta T = T_1 - T_2$.
- In each universe put either a resistor **or** an antenna and connect them with a perfect transmission line
- If $T_1 > T_2$, then power $P = 2k \cdot \Delta T \cdot B$ until the temperatures equalize.

  where k = Boltzman's constant = $1.38 \cdot 10^{-23}$ watts/°K/Hz

Power = Temperature

[and the Mysterious factor of 2]



- The receiver will see a delivered power

$$P = k \cdot T \cdot B$$

where the previous factor of 2 disappeared because the receiver only responds to half the noise signal.

The noise has half it's power in each of two orthogonal (i.e. sine vs cosine) components.

A second receiver, with its LO shifted by 90°, would see the other, independent component which also has a power

$$P = k \cdot T \cdot B$$

Flux Density & Janskys

- Flux Density is the measure of the amount of power falling on a 1 m^2 surface area.
- In radio astronomy, we measure the brightness of a radio source in Janskys:

$$1 \text{ Jansky} = 1 \text{ Jy} = 10^{-26} \text{ watts/m}^2\text{/Hz}$$

- In geodesy, most sources of interest have fluxes of 0.1 – 10 Jy
- A lot of high sensitivity astronomy is done on sources < 1 mJy (10^{-29} w/m^2/Hz)

Sensitivity

We saw earlier that a receiver will indicate the total power of $P = k \cdot T \cdot B$. Let's now consider what temperature contributions are in T:

T_{CMB} = cosmic microwave background ~ 3°K
+ T_{ATM} = atmosphere absorbs some of the signal
+ T_{LOSS} = antenna ohmic losses
+ $T_{SPILLOVER}$ = antenna feed sees trees, ground, etc
+ T_{LNA} = Low Noise Amplifier in receiver
+ T_{MISC} = other miscellaneous contributions

T_{SYS} = the sum of all these contributions

T_{SYS} must be compared to T_{SOURCE} = the small contribution from radio source that we want to see:

$$S/N = T_{SOURCE}/T_{SYS}$$

Sensitivity cont'd

- In radio astronomy, our signal is random noise. It can be shown that if we have noise with a Band-width B Hz, then we obtain a fresh, independent measurement of the noise every 1/B seconds.
- Now add all these independent samples for τ seconds (called the integration time).
- The total number of samples collected will be

$$n = B \cdot \tau$$

- Statistics tells us that the uncertainty of an average made up of n samples is $\approx 1/\sqrt{n}$

Sensitivity cont'd

- Therefore we see that our noise measurement will have an RMS noise level of

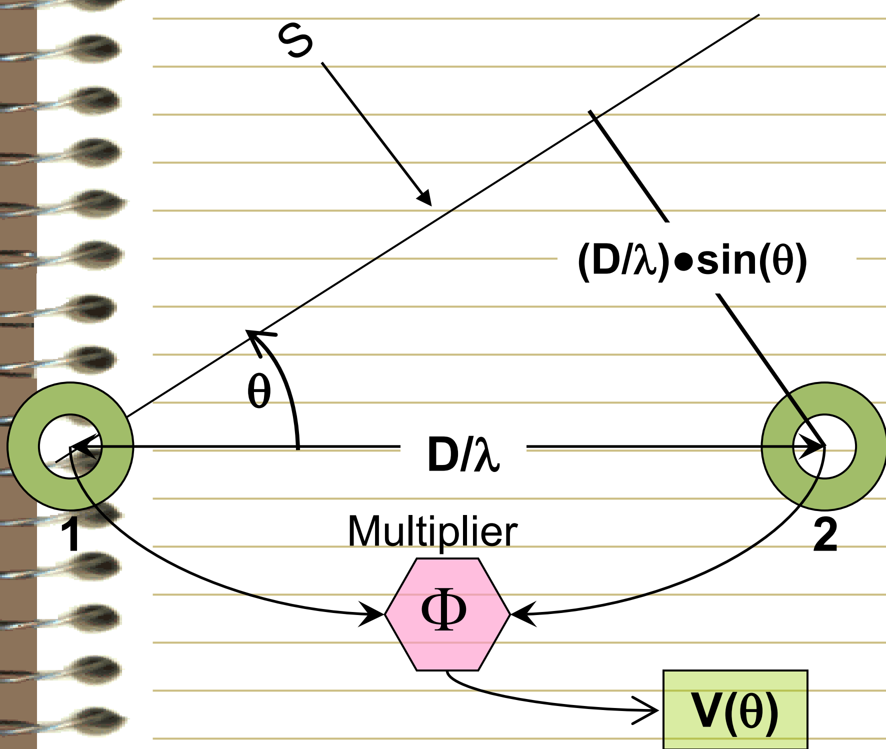
$$\delta T \approx \frac{T_{SYS} \cdot \rho}{\sqrt{(B \cdot \tau)}}$$

where ρ is a number typically in the range 1 to □ depending on the specifics of the radiometer

- To detect a source with good certainty, it is desirable to strive for $T_{SOURCE} > 5\ \delta T$
- This is usually achieved by
  - Integrate Longer to increase $\sqrt{\tau}$
  - Build a new receiver with better T_{SYS}
  - Use a bigger telescope

The Basic 2-Element Interferometer

Consider a 2 element interferometer with the elements separated by a distance **D** operating at a wavelength λ. Observe a distant source S at an angle θ. We bring the signals together & measure the phase $\Phi = [2\pi(\mathbf{D}/\lambda) \bullet \sin(\theta)]$ by multiplying the two signals.



Now let the source S move across the sky, changing the angle θ. The output of the multiplier (a.k.a. correlator) will be of the form:

$$V(\theta) \sim \cos(\Phi) = \cos[2\pi(D/\lambda) \sin(\theta)]$$

When the antenna is "broadside" with small θ we can use the approximation $\sin(\theta) \approx \theta$ and write

$$V(\theta) \sim \cos [2\pi(D/\lambda) \bullet \theta]$$

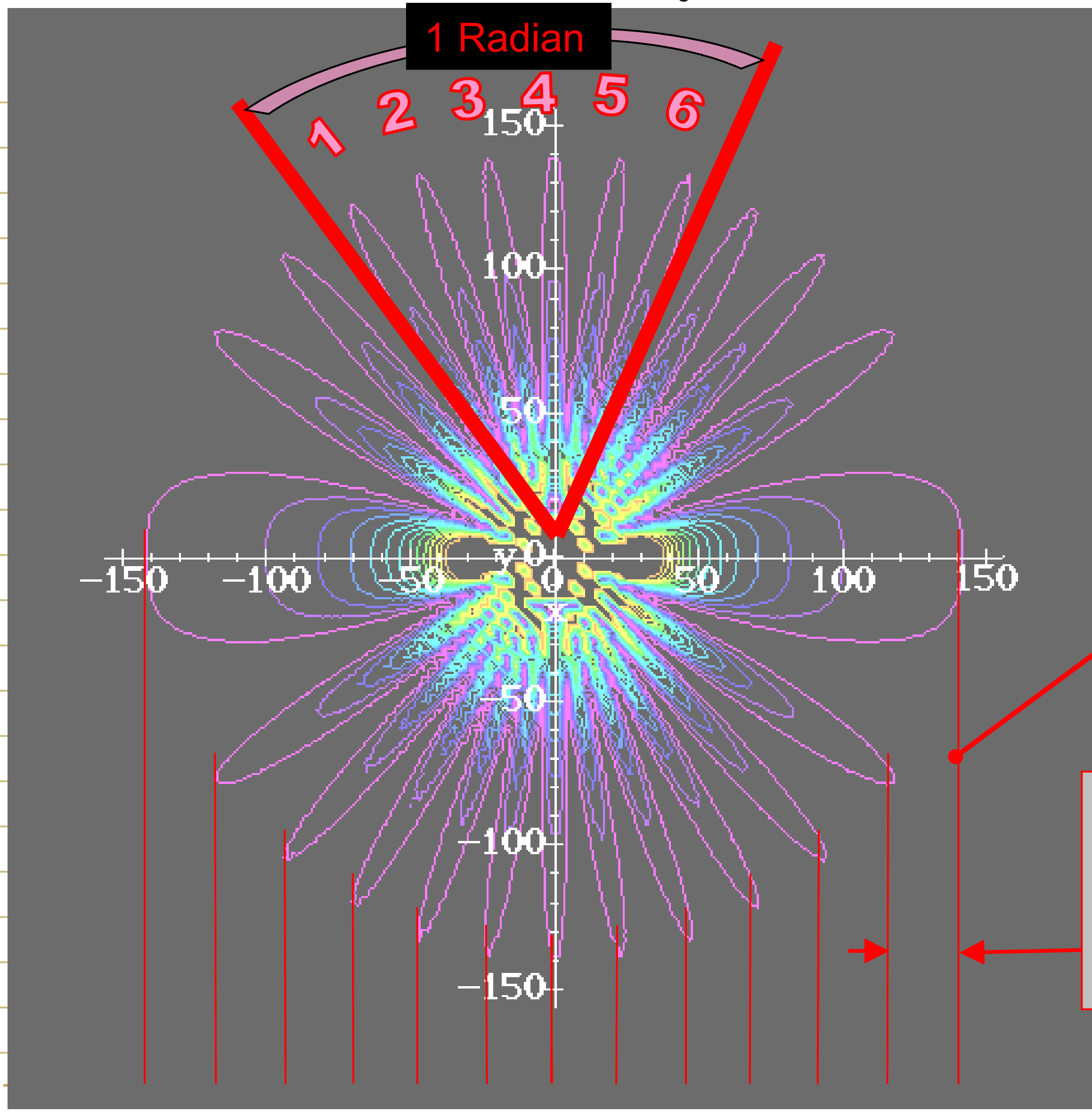
The Concept of Spatial Frequencies

- In the 2-element interferometer, we saw that the output is of the form $V(\theta) \sim \cos[2\pi(D/\lambda) \bullet \theta]$. We have sinusoidal "fringes" on the sky with a periodicity of

$$\Delta\theta \text{ (in radians)} = \lambda / D$$

- Example: Consider 2 small antennas spaced 4.2 meters apart at 70 cm are spaced by a distance of 6 λ, and will exhibit interferometer fringes spaced 1/6 radians = 9.5°.
- This expression for $V(\theta)$ is similar to the form of a cosine wave $V(t) \sim \cos(2\pi ft)$. We define the (spatial) frequency as (D/λ) and its orthogonal domain as θ, the angle on the sky (measured in radians). An interferometer acts as a filter to isolate structure of the sky that has a periodicity of λ / D cycles/radian
- If we have a complex pattern on the sky, then a series of baselines with different D/λ can be used to decompose the brightness distribution of the source. This is the basis of Aperture Synthesis which is the basis of much of modern Radio Astronomy!

Two antennas separated by 6λ

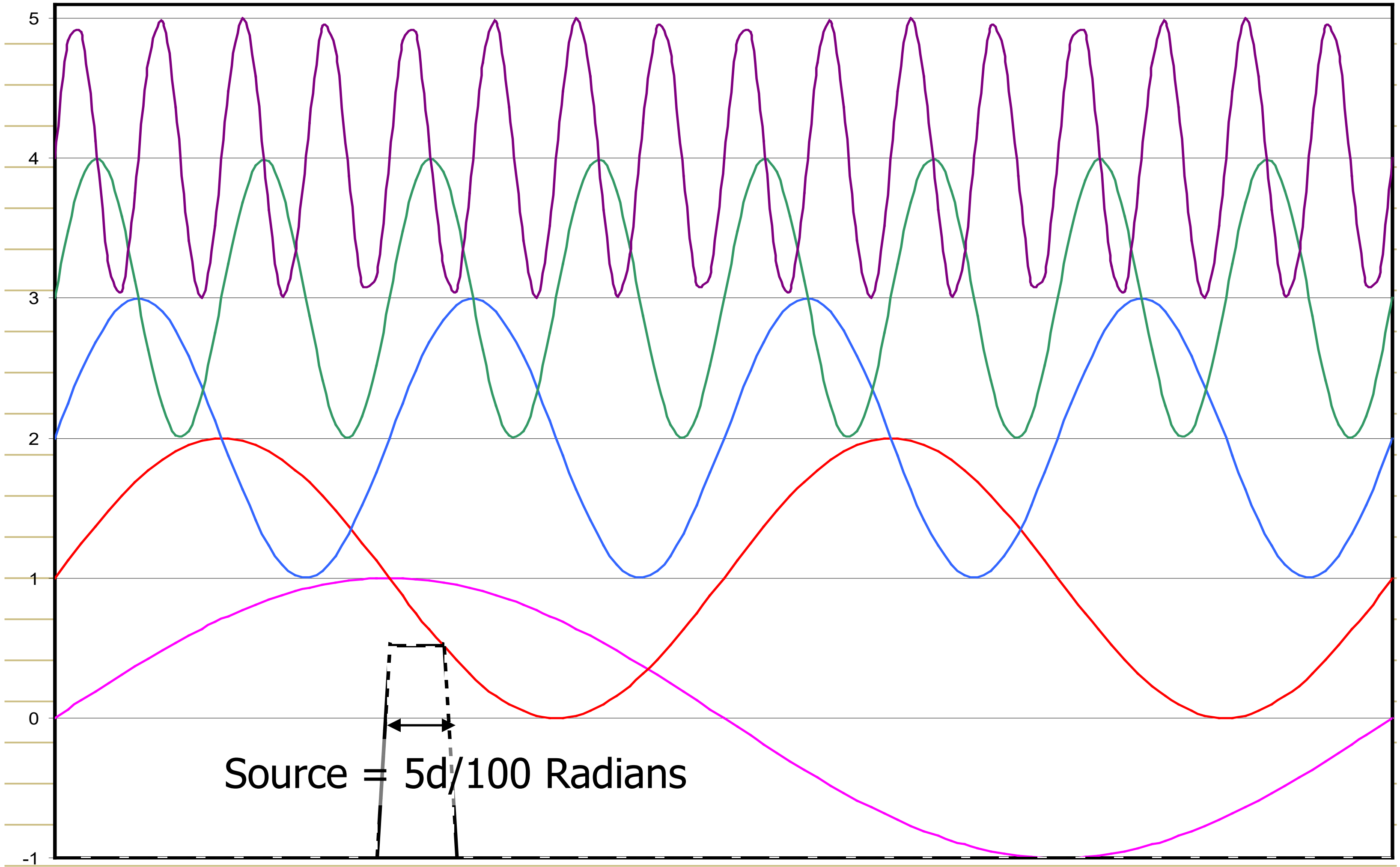


Equally Spaced "Bread Slices"

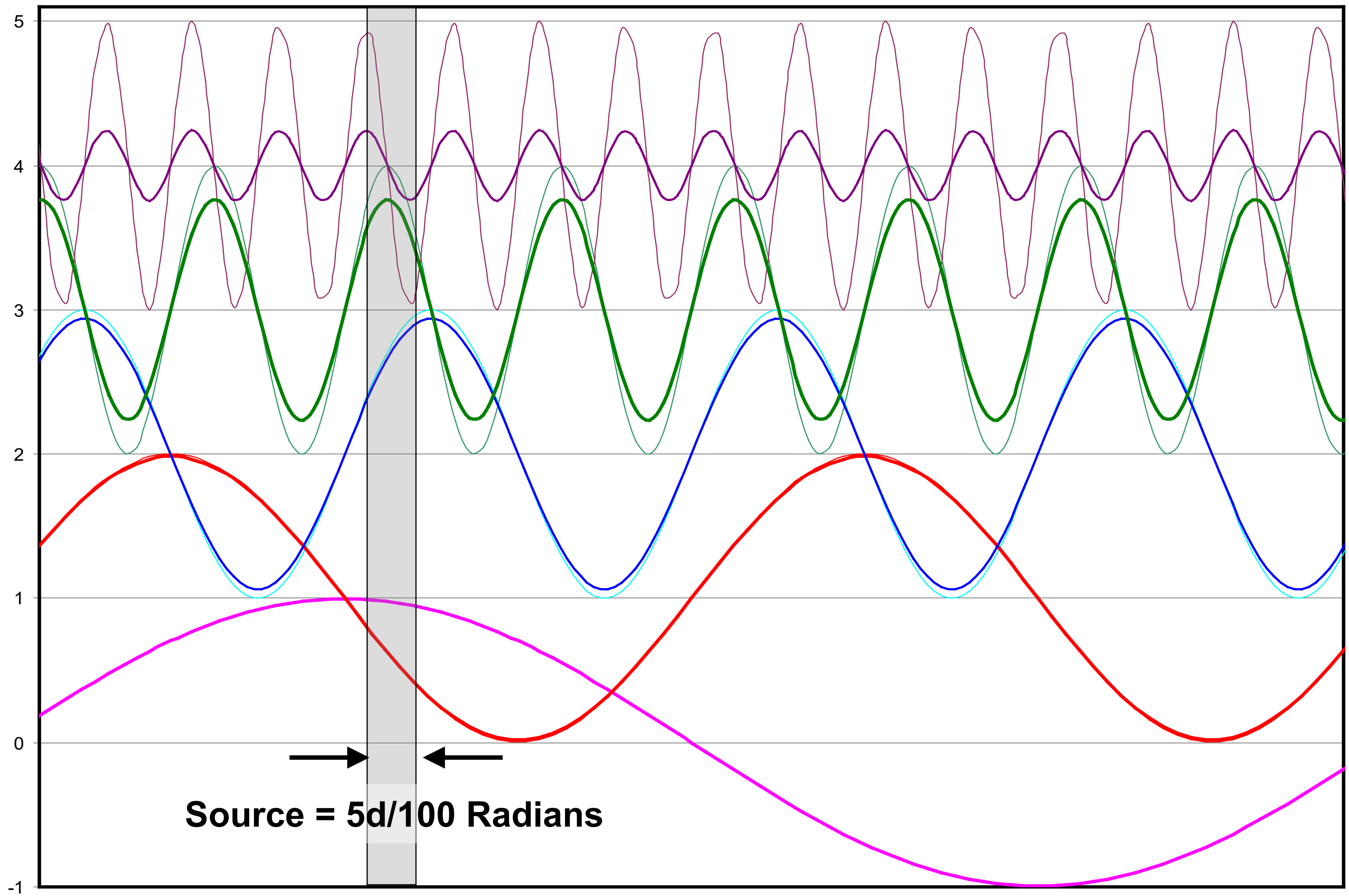
Spatial Frequency = D / λ cycles/radian

Some Interferometer Examples

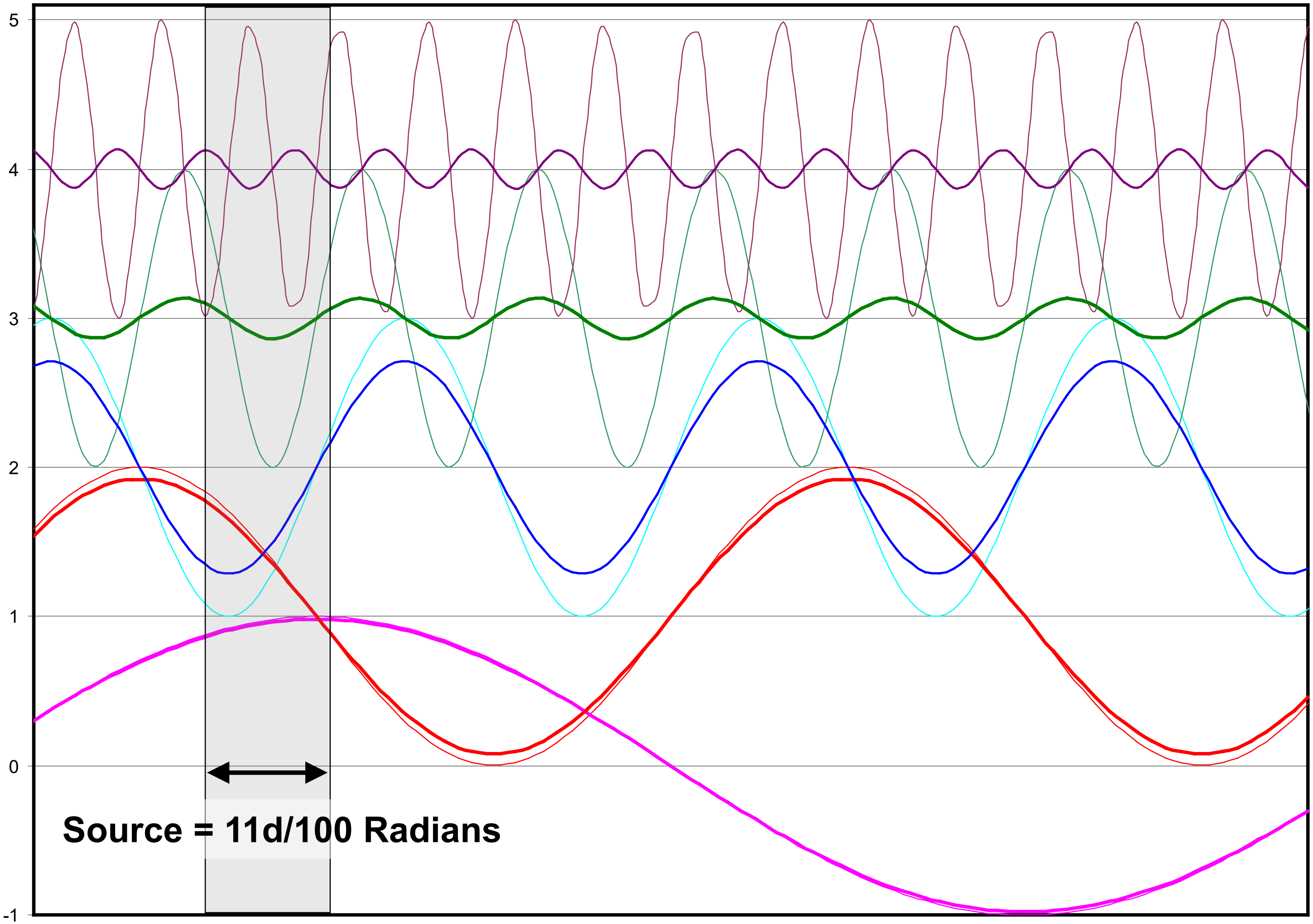
Interferometer fringes for D=d • (1,2,4,8,16)λ



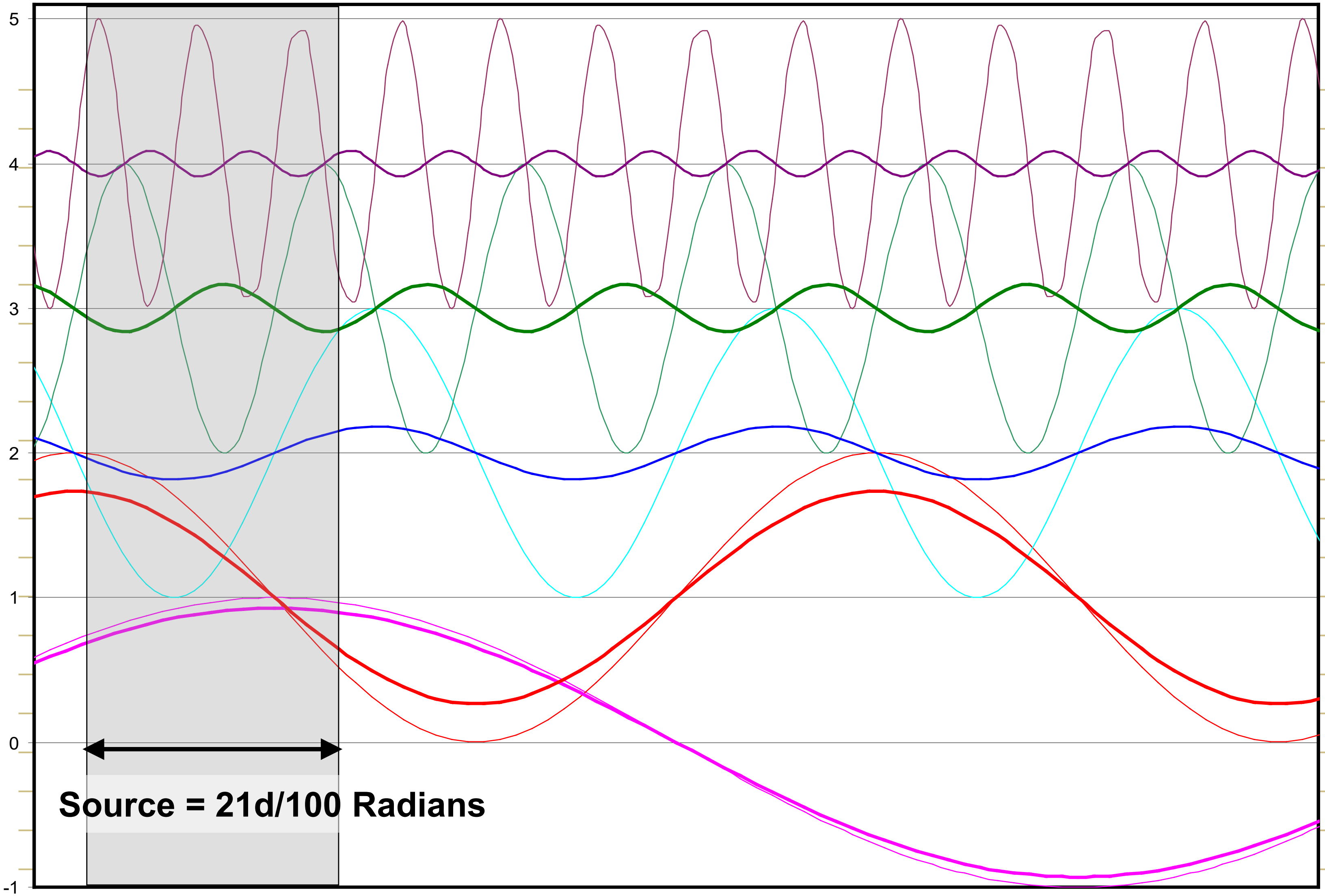
Resolution of source = 5•d/100 Radians



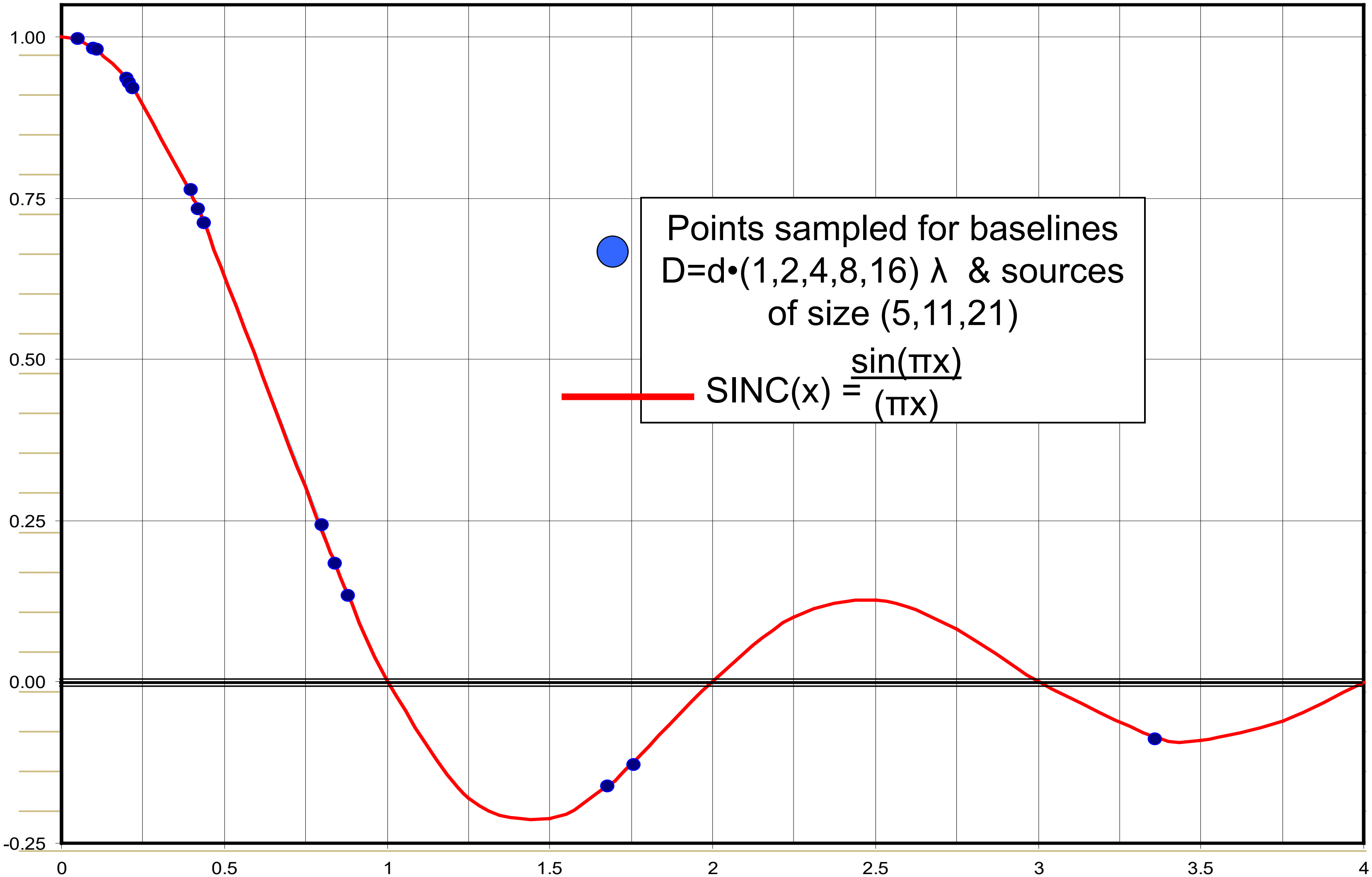
Resolution of source = 11.d/100 Radians



Resolution of source = 21•d/100Radians



Putting These Interferometers Together to make a One-Dimensional Visibility Function



A 27-element Interferometer
The VLA in New Mexico

The VLA consists of 27 85′ telescopes in a "Y" shape spanning a total of nearly 40 km west of Socorro, NM. (Sometimes a 28th element 52 km west of the VLA at Pie Town, NM is added for more resolution)

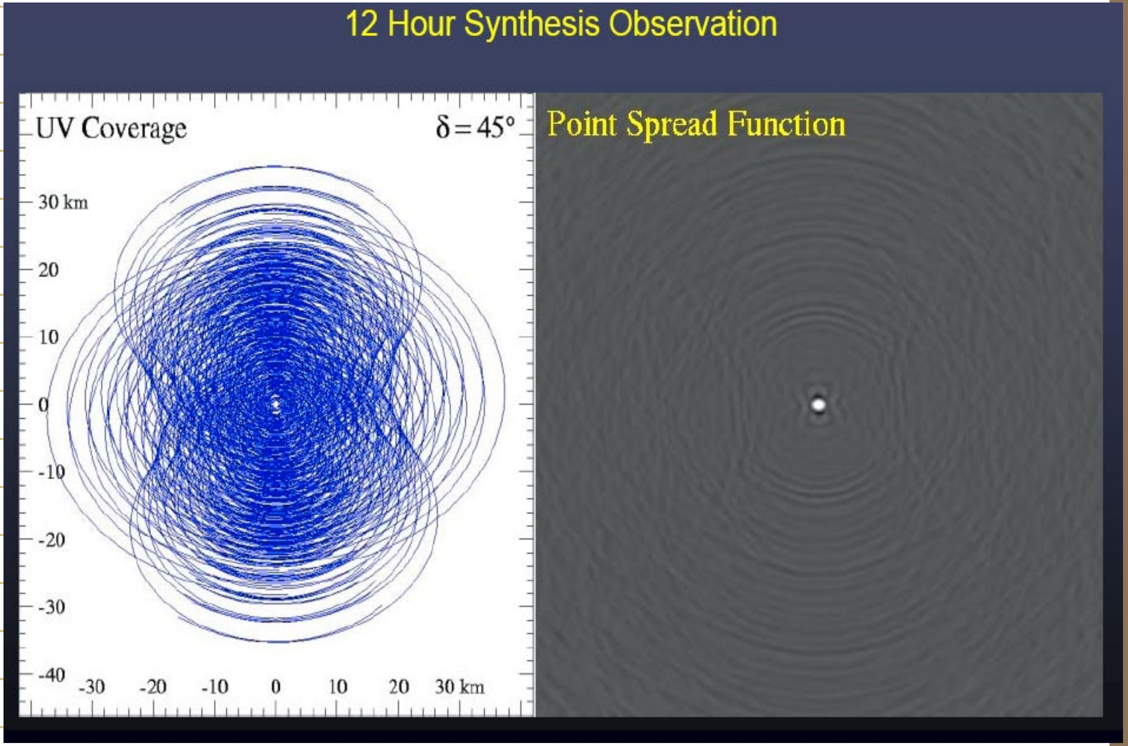
27 elements yield (27)*(26)/2 = 351 simultaneous 2 element interferometers.

2-Dimensional Spatial Frequencies

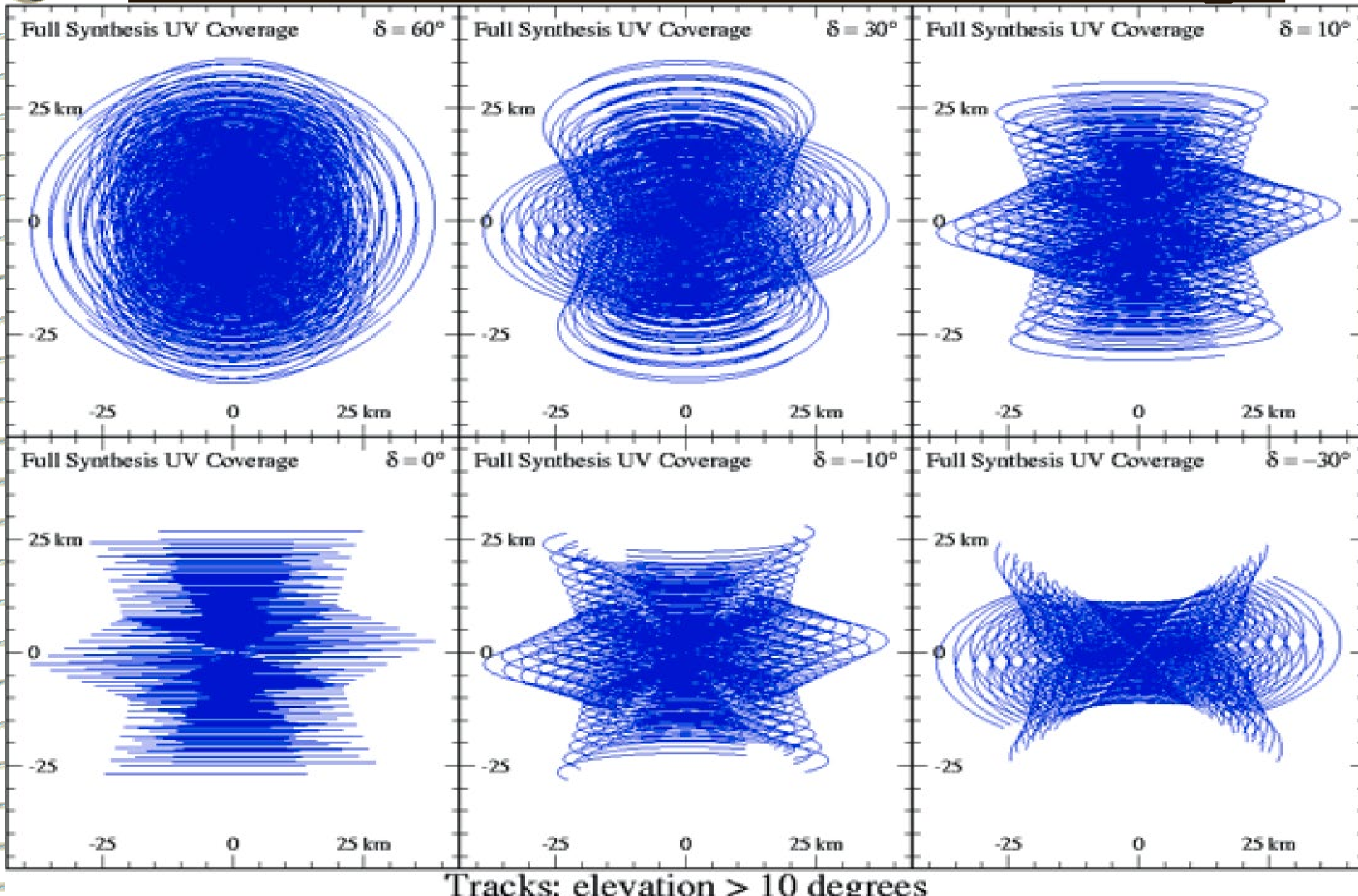
As the earth turns, the orientation of all 351 baselines rotate as seen from a source on the sky, synthesizing the equivalent of a ~40 km diameter dish:

This example shows the equivalent Aperture Synthesis "dish" formed by observing a source at δ=45° for 12 hours.

At the right, we see the "beam" formed by Fourier Transforming the U-V spatial coverage.

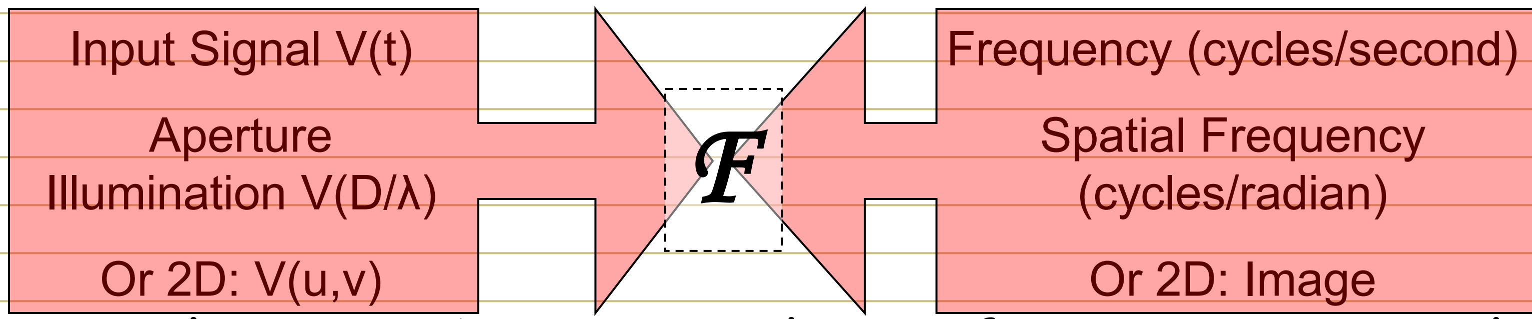


More VLA "u-v" Plane Coverage



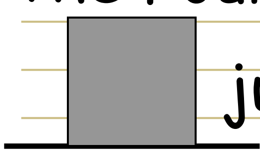
Fourier Transforms and Antennas

Just as frequency/time are related by a Fourier transform, the (voltage) distribution of signals across an antenna array is related to the (voltage) pattern of the antenna on the sky



Our earlier One-Dimensional interferometer example yielded a $\text{SINC}(X) = \frac{\sin(\pi x)}{(\pi x)}$ visibility function.

The Fourier Transform $\mathcal{F}$ {SINC(X)} is a "boxcar",



just like the d•(5,11,21)/100 model we assumed.

A Factoid to Remember

If we have two domains that are related by a Fourier Transform like we just described:

Input Signal V(t)

Aperture Illumination V(D/λ)

$\mathcal{F}$

Frequency (cycles/second)

Spatial Frequency (cycles/radian)

•If a "signal" is WIDE

in one domain, then it is Narrow in the other.

- •Big Antenna —> Small Beamwidth
- •Wide Beamwidth —> Small Antenna

$\Delta\theta$ (radians) $= \lambda / D$

- •Sharp Pulse —> Wide RF Bandwidth $\Delta f \approx 1/t$

Heisenberg in 1927.

QUANTUM MECHANICS 1925 - 1927

THE UNCERTAINTY PRINCIPLE

The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa.

--Heisenberg, uncertainty paper, 1927

$\Delta p \, \Delta q \geq h / 4\pi$

$\Delta E \, \Delta t \geq h / 4\pi$

"I knew of [Heisenberg's] theory, of course, but I felt discouraged, not to say repelled, by the methods of transcendental algebra, which appeared difficult to me, and by the lack of visualizability."

-*Schrödinger in 1926*

"The more I think about the physical portion of Schrödinger's theory, the more repulsive I find it...What Schrödinger writes about the visualizability of his theory 'is probably not quite right,' in other words it's crap. ! ! !

--*Heisenberg, writing to Pauli, 1926*

The Uncertainty Principle in The Real World

- What Schrödinger didn't understand is that Quantum Mechanics is intimately related to Fourier Transforms.
- One of the Heisenberg's two expressions for the uncertainty principle is

$$\Delta E \bullet \Delta t \geq h/2\pi \;,$$

where Planck's Constant h = 6.626×10^{-34} Joule-seconds

- We have learned that the change in Energy associated with an atomic transition between two levels ΔE is associated with the emission of a photon of frequency Δf as

$$\Delta E = h \bullet \Delta f.$$

- Substituting for ΔE & dividing by h, we get the equivalent expression for the Uncertainty Principle

$$\Delta f \bullet \Delta t \geq 1/2\pi$$

Some implications of $\Delta f \bullet \Delta t > 1/2\pi$

Measuring Frequency with a Counter

- If we measure the frequency of an oscillator with a counter, we count the number of cycles **N** that occur in a time **Δt** as defined by a clock in the counter.
- But because the **Δt** window can start & end anywhere in the sine wave, we have an uncertainty in the measurement of $\Delta N = \pm[0-2]$ counts. By averaging several measurements, we can determine $N \pm \Delta N$ to better than ±1 count.
- The oscillator's frequency is then determined to be

  $f = (N \pm \Delta N) / \Delta t$ with an uncertainty $\Delta f \bullet \Delta t \approx \pm 1$ count
- If we had actually measured the phase of the at the start and end of the Δt measurement, we could have achieved the Heisenberg limit $\Delta f \bullet \Delta t \approx 1/2\pi$ even if the S/N is poor.
- If the (S/N) is improved and the phase is measured more accurately, then the uncertainty will become

$$\Delta f \bullet \Delta t \approx 1/[2\pi \bullet (S/N)]$$

The Uncertainty Principle in Radio Astronomy

- Interferometers have an antenna pattern with sinusoidal peaks spaced $\Delta\theta=(\lambda/D)$ radians so measuring the position of a source to by counting fringes, we achieve a measurement precision of

  $\delta\theta \bullet (D/\lambda) \approx \pm 1$ fringe (just like the frequency counter).

- If the S/N is poor and we use fringe phase as the observable, then we reach the Uncertainty Principle limit of

$$\delta\theta \bullet (D/\lambda) \geq 1/2\pi$$

- If you have a reasonable (S/N), you can measure the phase of the sinusoidal fringe and infer the position with an uncertainty $\delta\theta$ of fraction of a radian:

$$\delta\theta \approx (\lambda/D) \bullet (S/N)^{-1} \bullet 1/2\pi$$

- For an intermediate VLA baseline (~8km) @ 15 GHz (λ=2cm) we have fringes spaced $\Delta\theta=\lambda/D = 5\times10^{-7}$ radians = $\frac{1}{2}$ arcsecond and it should be possible to measure source positions to $\delta\theta <$ .02 arc sec, assuming all measurement errors (including atmospheric path delays) can be calibrated.
- VLBI baselines as long as 12,000 km (Hawaii to South Africa) at λ=3.8 cm. ($D\sim3\times10^{8}\,\lambda$) yield fringes of $\Delta\theta<1$ milliarcsecond in size.

Properties of the Fourier Transform
(or, Fourier's Song)

Integrate your function times a complex exponential.
It's really not so hard, you can do it with your pencil.
And when you're done with this calculation
You've got a brand new function - the Fourier Transformation.
What a prism does to sunlight, what the ear does to sound,
Fourier does to signals, it's the coolest trick around.
Now filtering is easy, you don't need to convolve;
all you do is multiply in order to solve.

From time into frequency --- from frequency to time

Every operation in the time domain,
has a Fourier analog - that's what I claim.
Think of a delay, a simple shift in time –
It becomes a phase rotation - now that's truly sublime!
And to differentiate, here's a simple trick,
just multiply by j omega, ain't that slick?
Integration is the inverse, what you gonna do?
Divide instead of multiply - you can do it too.

From time into frequency --- from frequency to time

Let's do some examples... consider a sine.
It's mapped to a delta, in frequency - not in time.
Now take that same delta as a function of time,
Mapped into frequency - of course - it's just a sine!
Sine x on x is handy, let's call it a sinc.
Its Fourier Transform is simpler than you think.
You get a pulse that's shaped just like a top hat...
Squeeze the pulse thin, and the sinc grows fat.
Or make the pulse wide, and the sinc grows dense,
The uncertainty principle is just common sense. –

Stolen from Bill Sethares @ http://eceserv0.ece.wisc.edu/~sethares/mp3s/fourier.html

Quasars and other beasties

- In the mid 1960's it was noted that some "radio stars" were variable on time scales ~weeks to months. It is hard to envision any source that is larger than ~1 light-month in size that can vary that rapidly.
- For one of these "quasars" (3C273), old photographic plates (like from 1929) tell that the optical "star" is also variable.
- If we assume that these object are extragalactic, then the sources must have sizes and/or structure measured in milliarcseconds (i.e. ~10^{-8} radians).
- If the sources are this small and this far away, then the equivalent brightness temperatures must be ~ 10^{14} to 10^{15} °K !!!

Quasars and other beasties

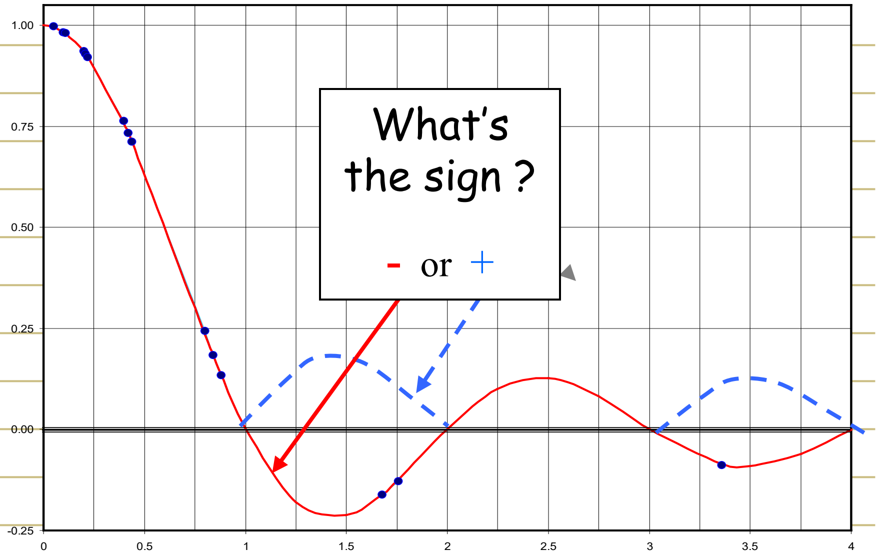
- In order to measure the size a source 10^{-8} radians in size, we need a baseline ≥ $10^8\lambda$.
- At a wavelength λ = 10cm, this requires baselines ≥ 10^7m = 10^4 km 6000 miles.
  - [an aside: The meter was originally defined as 10^{-7} times the distance from pole to equator along the meridian of Paris. This leads to the circumference of the earth ≈ 40,000 km and the radius of the earth ≈ 40,000/2π = 6370 km]
- In 1967, groups in the US and Canada succeeded in breaking the 1000 km barrier using atomic clocks and tape recorders.
  - US = Mark-1 with 800 BPI 7-track computer tape (360 kHz, 720 kb/s, with one tape lasting 3 minutes): Greenbank-Arecibo = 2550 km @ 610MHz = 5.2 Megaλ => 38 milliarcsec fringes.
  - Canada = Analog studio video tape recorders (4 Mhz): Algonquin-Penticton = 3074 km @ 448 MHz = 4.6 Megaλ => 44 milliarcsec fringes.
- After about 1968, all systems migrated to digital recording using Computer Tape (Mk1 & DSN), Video Tape (Mk2, Canada, Japan), Instrumentation Tape (Mk3 & 4) and now RAID-like Computer Disk Arrays.
- By 1971 well-sampled visibility curves of 3C279 showed a well defined double source
  - Haystack-Goldstone baseline @ λ=3.8 cm (100 Megaλ => 2 milliarcsec fringes.
  - These measurements were repeated a few months later and showed apparent superluminal motion (velocity ≈ 10c).

Galactic & Solar System Objects

- Also in 1967 (with Mark-1) were the first observations of OH Masers at 1665-1667 MHz (λ = 18 cm). These objects exhibit numerous small, narrow bandwidth "hot spots".
- Later, other Maser sources associated with methanol, H_2O, SiO, NH_3 and other chemicals have been detected.
- At frequencies below ~ 1 GHz pulsars have proven interesting.
- The planet Jupiter radiates "bursts" at frequencies below 38MHz. VLBI on Jupiter dates back to the early 1960's, predating the Quasar VLBI!
- Interplanetary spacecraft have been tracked with VLBI, using differential measurements between the spacecraft and quasars for navigation.
- The Apollo "Lunar Rover" was tracked (λ = 13 cm) with respect to the LEM "home base".

Phase in Interferometry

- We noted earlier that the image of a source observed by an interferometer array can be related to the observed visibility function via a Fourier transform.



- This assumes that each data point is a **complex** phase (& sign) and amplitude.
- In VLBI, we have independent phase/frequency standards (H-masers), so we have lost track of the absolute RF signal phase

Phase in VLBI (1)

VLBI people have come up with 3 main ways to solve the undefined phase dilemma:

1. Rapidly switch between the source of interest and a nearby "point" source. Then do the mapping W.R.T. the reference source. The sources need to be close enough so that phase errors caused by the atmosphere are the same.
   - If you are really lucky, the reference source is in the same telescope field of view. This has been used extensively for mapping of OH, H_2O etc. maser sources.
   - The switching needs to be fast enough so that phase drifts in the H-Maser and atmosphere are small.

Phase in VLBI (2)

2. If we observe a source with 3 (or more) stations with different baselines, then we can use "**closure phase**".
   - If the source is symmetric then the 3 observed phases $\Phi_{AB} + \Phi_{BC} + \Phi_{CA} = 0$.
   - If the source is not symmetric (like a core+jet), then the triplet phase $\neq 0$.
   - Models containing the observed closure phases and amplitudes can be "observed" in the computer and iterated until the observations from the model match the data.
   - Then the paper is sent to *Ap.J.*

Phase in VLBI (3)

3. Especially for Geodesy and Astrometry, the principal observation type is called the

   "Group Delay" = $\tau_G = \Delta\Phi_f / \Delta f$.

   - Usually, fringe phase Φ_f is measured in a series of separated, narrow bands (at IF) that cover a wider "spanned bandwidth" in a technique named "bandwidth synthesis". A common example is the use of 8 IF channels at X-band spanning more than 700 MHz.
   - We earlier saw that the Uncertainty Principle predicts $\Delta f \bullet \Delta t \approx 1/[2\pi\bullet(S/N)]$. Therefore a spanned bandwidth Δf~500 MHz & S/N~30 would have an RMS uncertainty ~11 picoseconds.

FINIS

Thank you for participating in this marathon!

Any Questions (or are you ready for coffee?)